RELATIVISTIC QUANTUM THEORY OF PARTICLES WITH VARIABLE MASS, II

By H. C. Corben

SPACE TECHNOLOGY LABORATORIES, INC., REDONDO BEACH, CALIFORNIA

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Introduction.—In the first part of this paper,[1] it was shown that the following wave equation offers a new method for describing a free particle in relativistic quantum theory:

$$(i\epsilon_i p_i + Mc)\psi = 0, \tag{1}$$

where M is the operator

$$M = m + m_0\epsilon_{kl}\eta_{kl} + m'\epsilon_k\eta_k. \tag{2}$$

Here, m, m_0, m' are any scalar hermitian operators which commute with ϵ_{kl}, η_{kl}, p_i, and in particular they may be c-numbers. The momentum operators p_i also commute with ϵ_i, ϵ_{kl}, η_i, η_{kl}. The operators η_i, η_{kl} commute with the operators ϵ_i, ϵ_{kl}, and these satisfy among themselves the following commutation relations:

$$(\epsilon_{ij}, \epsilon_k) = \epsilon_i\delta_{jk} - \epsilon_j\delta_{ik}, \tag{3}$$

$$(\eta_{ij}, \eta_k) = \eta_i\delta_{jk} - \eta_j\delta_{ik}, \tag{4}$$

$$(\epsilon_{ij}, \epsilon_{kl}) = \epsilon_{il}\delta_{jk} + \epsilon_{jk}\delta_{il} - \epsilon_{ik}\delta_{jl} - \epsilon_{jl}\delta_{ik}, \tag{5}$$

$$(\eta_{ij}, \eta_{kl}) = \eta_{il}\delta_{jk} + \eta_{jk}\delta_{il} - \eta_{ik}\delta_{jl} - \eta_{jl}\delta_{ik}. \tag{6}$$

If equation (1) is written in the form $H\psi = 0$, the condition for conservation of total angular momentum,

$$(S_{ij} + x_ip_j - x_jp_i, H) = 0, \tag{7}$$

is then satisfied if the spin operator S_{ij} is given by

$$S_{ij} = -i\hbar(\epsilon_{ij} + \eta_{ij}). \tag{8}$$

The charge-current four-vector,

$$j_i = \psi^*\chi\epsilon_i\psi, \tag{9}$$

is then found to be conserved if χ is an operator which anticommutes with ϵ_1, ϵ_2, ϵ_3 and commutes with ϵ_4, p_i and M.

Although equations (1) and (9) may be applied to a particle of arbitrary integral or half-integral spin and arbitrary integral charge, we restrict our attention to the special case

$$\left.\begin{aligned}\epsilon_{ij} &= \epsilon_i\epsilon_j - \epsilon_j\epsilon_i\\ \eta_{ij} &= \eta_i\eta_j - \eta_j\eta_i\end{aligned}\right\}, \tag{10}$$

in which the spin is required to be 0, $^1/_2$, 1, $^3/_2$, or 2 and the charge operator $-i\chi\epsilon_{4/c}$ has eigenvalues $\pm e_0$ or 0. In this case, equations (5) and (6) follow from (3) and (4). It then follows that

$$\left.\begin{aligned} \epsilon_i &= {}^1/_2\gamma_i \text{ or } \beta_i \\ \eta_i &= {}^1/_2\gamma_i' \text{ or } \beta_i' \end{aligned}\right\}, \tag{11}$$

where γ_i, γ_i' satisfy the Dirac commutation relations

$$\left.\begin{aligned} \gamma_i\gamma_j + \gamma_j\gamma_i &= 2\delta_{ij} \\ \gamma_i'\gamma_j' + \gamma_j'\gamma_i' &= 2\delta_{ij} \end{aligned}\right\}, \tag{12}$$

and the β_i, β_i' satisfy the Kemmer-Duffin relations

$$\left.\begin{aligned} \beta_i\beta_j\beta_k + \beta_k\beta_j\beta_i &= \delta_{ij}\beta_k + \delta_{kj}\beta_i \\ \beta_i'\beta_j'\beta_k' + \beta_k'\beta_j'\beta_i' &= \delta_{ij}\beta_k' + \delta_{kj}\beta_i' \end{aligned}\right\}, \tag{13}$$

the γ_i, β_i commuting with the γ_i', β_i'.

In this paper, we consider the special case of equation (2) for which $m' = 0$, and in which m and m_0 are arbitrary c-number parameters. We are therefore led to the following four generalizations of the Dirac and Kemmer-Duffin equations:

$$\left.\begin{aligned} &(i\gamma_i p_i + m_1 + m_1^0\gamma_{kl}\gamma'_{kl})\psi = 0, \\ &S_{ij} = -\tfrac{i\hbar}{4}(\gamma_{ij} + \gamma'_{ij}), \qquad j_i = ie_0c\psi^*\gamma_4\gamma_4'\gamma_i\psi \end{aligned}\right\}; \tag{14}$$

$$\left.\begin{aligned} &(i\gamma_i p_i + m_2 + m_2^0\gamma_{kl}\beta_{kl})\psi = 0, \\ &S_{ij} = -i\hbar\,({}^1/_4\gamma_{ij} + \beta_{ij}), \qquad j_i = ie_0c\psi^*\gamma_4\mu_4\gamma_i\psi \end{aligned}\right\}; \tag{15}$$

$$\left.\begin{aligned} &(i\beta_i p_i + m_3 + m_3^0\beta_{kl}\gamma_{kl})\psi = 0, \\ &S_{ij} = -i\hbar\,(\beta_{ij} + {}^1/_4\gamma_{ij}), \qquad j_i = ie_0c\psi^*\gamma_4\mu_4\beta_i\psi \end{aligned}\right\}; \tag{16}$$

$$\left.\begin{aligned} &(i\beta_i p_i + m_4 + m_4^0\beta_{kl}\beta'_{kl})\psi = 0, \\ &S_{ij} = -i\hbar\,(\beta_{ij} + \beta'_{ij}), \qquad j_i = ie_0c\psi^*\mu_4\mu_4'\beta_i\psi \end{aligned}\right\}; \tag{17}$$

where $\mu_4 = 2\beta_4^2 - 1$, $\mu_4' = 2\beta_4'^2 - 1$. The charge-current four-vectors listed in the various cases are seen to be conserved since μ_4 anticommutes with β_1, β_2, and β_3 and commutes with β_4.

Bosons.—We note that equation (14) may be separated in a manner similar to that used in the two-component neutrino theory:

$$(i\gamma_i p_i + m_1 + m_1^0\gamma_{kl}\gamma'_{kl})(1 \pm \gamma_5')\psi = 0.$$

Some of the properties of the solutions of this equation in the system in which $\mathbf{p} = 0$ have been discussed in reference 1. Writing $\gamma_i = \rho_2\,\boldsymbol{\sigma}$, $\gamma_i' = \tau_2\,\Sigma$ $(i = 1, 2, 3)$, $\gamma_4 = \rho_3$, $\gamma_4' = \tau_3$ and using the usual representation for the γ_i and a similar representation for the γ_i', we find that ψ can be represented as a sixteen-component spinor separable, as indicated above, into eight-component spinors which describe a particle of spin zero or unity with rest energy given by

$$W_0 = \pm c^2\sqrt{m_1(m_1 + 48m_1^0)} \quad \text{spin zero} \quad (\uparrow\downarrow),$$

$$W_1 = \pm c^2\sqrt{m_1(m_1 - 16m_1^0)} \quad \text{spin one.} \quad (\uparrow\uparrow).$$

The rest mass operator is hermitian, but since it does not commute with $\gamma_4 = \rho_3$, it follows that the rest-energy operator is not hermitian and its eigenvalues are not required to be real. If $16\,m_1^0 > m_1 > 0$, the spin-one state is in fact highly unstable. With appropriate choice of the parameters m_1, m_1^0 (e.g. $m_1 = m_e$, $m_1^0 = 1{,}555m_e$) it follows that the spin zero state would have the rest energy of the $\pi^\pm$ meson.

Although it is in an eigenstate of rest-energy, its rest mass would fluctuate between m_1 and $m_1 + 48\, m_1^0$ (particle of Salam and Ward?[2]). The spin zero-state is given in the rest system by

$$\begin{pmatrix} \alpha - 1, & 2, & -1, & 2 \\ 2, & \alpha - 1, & 2, & -1 \\ 1, & -2, & \beta + 1, & -2 \\ -2, & 1, & -2, & \beta + 1 \end{pmatrix} \begin{pmatrix} \psi_2 \\ \psi_3 \\ \psi_{14} \\ \psi_{15} \end{pmatrix} = 0, \tag{18}$$

where $\alpha = \dfrac{W_0 - m_1c^2}{8m_1{}^0c^2}$, $\beta = \dfrac{W_0 + mc_1{}^2}{8m_1{}^0c^2}$, $\alpha\beta + 3\alpha - 3\beta = 0,$

together with the equations obtained by replacing α by $-\beta$, β by $-\alpha$, i.e., by replacing W_0 by $-W_0$. A normalized eigenstate with eigenvalues for S_z, S^2 equal to zero ($\boldsymbol{\sigma}\cdot\Sigma = -3$) is then

$$\psi = \psi_{0,0} = \frac{1}{2\sqrt{m_\pi{}^2 + m_1{}^2}} \begin{pmatrix} m_\pi + m_1 \\ -(m_\pi + m_1) \\ -(m_\pi - m_1) \\ (m_\pi - m_1) \end{pmatrix} \exp\ [-im_\pi c^2t/\hbar],$$

where we have written $W_0 = m_\pi c^2$.

The charge density is given by

$$\rho = -ij_4/c^2 = e_0\psi^*\tau_3\psi,$$

and since in this subspace

$$\tau_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

we have

$$\rho = e_0 \frac{2m_1m_\pi}{m_1{}^2 + m_\pi{}^2}\, \psi^*\psi.$$

The state $W_0 = -m_\pi c^2$ then leads to a charge density of the opposite sign.

For the spin-one state ($\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma} = 1$), solutions in the rest system are given by (18), for $S_z = 0$, with

$$\alpha = \frac{iq - m_1}{8m_1{}^0},\quad \beta = \frac{iq + m_1}{8m_1{}^0},\quad \alpha\beta + \beta - \alpha = 0,$$

together with solutions in which q has been replaced by $-q(q = \sqrt{m_1(16m_1{}^0 - m_1)} \doteq m_\pi/\sqrt{3}$ for $m_1 \ll m_1{}^0)$. Similarly, eigenvalues $S^2 = 2\hbar^2$, $S_z = \pm\hbar$ characterize states

$$\psi_{1,\pm 1} = \begin{pmatrix} m_1 + iq \\ m_1 - iq \end{pmatrix} \exp\ (qc^2t/\hbar).$$

These states represent rapidly growing or damped waves and are not normalizable. This is consistent with the fact that probability is not conserved in this theory. However, we find that for each of these states $\psi^*\tau_3\psi = 0$, so that these damped or growing solutions do not violate charge conservation.

The spin-zero and spin-one states discussed above are both coupled and modified by a real or virtual electromagnetic field, causing the otherwise stable spin-zero state, tentatively identified above with the $\pi^{\pm}$ meson, to be quenched by coupling to the highly unstable state of spin-one. Although details of this process must await the second quantization of the theory, it is clear that the decay products from the eight-component spin-one state must be a Dirac particle and a neutrino which emerge with their spins parallel.

If we set $m_1 = 0$ in the above analysis, we find that the particle is stable, with spin 0 or 1 and with zero rest-mass and charge. For this case, equation (14) therefore leads to an alternative description of the photon.[3]

Other bosons of spin 0, 1, 2 are described by equation (17), but their properties are much more complicated. Such particles could decay only into other bosons, but if one of these decay products were highly unstable, such as the spin-one state discussed above, it would decay immediately into two fermions.

Fermions.—We now examine equation (15) in the rest-system of the particle. It is convenient to introduce the previous notation for the γ_i and to write for the β_{kl}

$$\begin{aligned} \mathbf{\Sigma} &= -i(\beta_{23}, \beta_{31}, \beta_{12}), \\ \boldsymbol{\lambda} &= -i(\beta_{14}, \beta_{24}, \beta_{34}), \\ \boldsymbol{\beta} &= (\beta_1, \beta_2, \beta_3). \end{aligned}$$

The spin of the particle is therefore

$$\mathbf{S} = \hbar(^1/_2\boldsymbol{\sigma} + \mathbf{\Sigma}),$$

where

$$\mathbf{\Sigma} \times \mathbf{\Sigma} = i\mathbf{\Sigma} = \boldsymbol{\lambda} \times \boldsymbol{\lambda},$$

$$(\boldsymbol{\sigma}\cdot\mathbf{\Sigma})^2 + (\boldsymbol{\sigma}\cdot\mathbf{\Sigma}) - \Sigma^2 = 0,$$

$$\Sigma^2 = \beta^2(3 - \beta^2).$$

Since

$$S^2 = \hbar^2(^3/_4 + \Sigma^2 + \boldsymbol{\sigma}\cdot\mathbf{\Sigma}),$$

it follows that a spin $^1/_2$ particle is characterized by the eigenvalues

$$\Sigma^2 = 2, \quad \boldsymbol{\sigma}\cdot\mathbf{\Sigma} = -2, \quad \beta^2 = 1, 2\ (\uparrow \Downarrow)$$

or

$$\Sigma^2 = 0, \quad \boldsymbol{\sigma}\cdot\mathbf{\Sigma} = 0, \quad \beta^2 = 0, 3.(\uparrow .)$$

Similarly, a particle of spin $^3/_2$ is characterized by

$$\Sigma^2 = 2, \quad \boldsymbol{\sigma}\cdot\mathbf{\Sigma} = 1, \quad \beta^2 = 1, 2.(\uparrow \Uparrow)$$

In the rest system of the particle, the energy operator of equation (15) may now be written

$$\frac{W}{c^2}\psi = [\rho_3(m_2 - 4m_2{}^0\boldsymbol{\sigma}\cdot\mathbf{\Sigma}) - 4im_2{}^0\rho_2\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}]\psi. \tag{19}$$

The fourth component of the conserved four-vector j_i is then

$$j_4 = ice_0\psi^*(2\beta_4{}^2 - 1)\psi.$$

The simplest representation of the β_i is that in which $\beta_i = 0$. In this case, equation (15) reduces to the Dirac equation for the electron if we set $m_2 = m_e$ and to the equation for the free neutrino if we set $m_2 = 0$.

On squaring (19), we have

$$\frac{W^2}{c^4}\psi = [m_2^2 - 8m_2m_2^0\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma} + 16(m_2^0)^2\{(\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma})^2 - (\boldsymbol{\sigma}\cdot\boldsymbol{\lambda})^2 + \rho_1(\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}, \boldsymbol{\sigma}\cdot\boldsymbol{\lambda})\}]\psi. \quad (20)$$

In computing the eigenvalues of this operator, relations such as the following may be used:

$$\begin{gathered}(\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}, \boldsymbol{\sigma}\cdot\boldsymbol{\lambda}) = -2[\beta^2\boldsymbol{\sigma}\cdot\boldsymbol{\lambda} + i(2\beta^2 - 1)\omega],\\ \omega = (\boldsymbol{\sigma}\cdot\boldsymbol{\beta})\beta_4,\ (\beta^2, \omega) = \omega,\ (\beta_4^2, \omega) = -\omega,\\ (\beta^2, \boldsymbol{\sigma}\cdot\boldsymbol{\lambda}) = -(\beta_4^2, \boldsymbol{\sigma}\cdot\boldsymbol{\lambda}) = -(2i\omega + \boldsymbol{\sigma}\cdot\boldsymbol{\lambda}),\\ (\beta_4, \boldsymbol{\sigma}\cdot\boldsymbol{\beta}) = -i\boldsymbol{\sigma}\cdot\boldsymbol{\lambda},\ (\beta_4, \boldsymbol{\sigma}\cdot\boldsymbol{\lambda}) = i\boldsymbol{\sigma}\cdot\boldsymbol{\beta}.\end{gathered}$$

However, it is much simpler to use the Kemmer representations,[4] from which one obtains the basic representation of $\boldsymbol{\Sigma}$:

$$\Sigma_x = \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & -i\\ 0 & i & 0\end{pmatrix},\quad \Sigma_y = \begin{pmatrix}0 & 0 & i\\ 0 & 0 & 0\\ -i & 0 & 0\end{pmatrix},\quad \Sigma_z = \begin{pmatrix}0 & -i & 0\\ i & 0 & 0\\ 0 & 0 & 0\end{pmatrix}.$$

In the 5×5 representation, these appear surrounded by two rows and columns of zeros, and in the 10×10 representation, they appear three times on the diagonal, the last row and column being zero. The operators $\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}$, $\boldsymbol{\sigma}\cdot\boldsymbol{\lambda}$ are then represented by matrices with 10 or 20 rows and columns, and ψ in equation (20) by a vector with 20 or 40 elements.

Some of these operators are diagonal in this representation, and their eigenvalues are given in Table 1. Here the symbol A stands for rows and columns 1, 2, 3, or 11, 12, 13 of the 20×20 matrix, B for 4, 5, 6, 14, 15, 16, C for 7, 8, 9, 17, 18, 19, and D for 10, 20, with a similar interpretation for E, F, G.

TABLE 1

EIGENVALUES OF CERTAIN OPERATORS FOR SPIN 1/2 AND SPIN 3/2 PARTICLES

Representation of the β_i	Row and column	Spin	$\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}$	Σ^2	λ^2	$(\boldsymbol{\sigma}\cdot\boldsymbol{\lambda})^2$	$\boldsymbol{\beta}^2$	$(\boldsymbol{\sigma}\cdot\boldsymbol{\beta})^2$	$2\beta_4^2-1$
10×10	A	1/2	−2	2	2	4	1	3	1
	B	1/2	−2	2	2	4	2	4	−1
	C	1/2	−2	2	1	3	2	4	1
	D	1/2	0	0	3	3	3	3	−1
5×5	E	1/2	0	0	3	3	0	0	1
	F	1/2	−2	2	1	3	1	3	−1
	G	1/2	0	0	0	0	3	3	1
10×10	A	3/2	1	2	2	1	1	0	1
	B	3/2	1	2	2	1	2	1	−1
	C	3/2	1	2	1	0	2	1	1
5×5	F	3/2	1	2	1	0	1	0	−1

We note that for A and B, $(\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma})^2 = (\boldsymbol{\sigma}\cdot\boldsymbol{\lambda})^2$, and it may be shown also in this case that $(\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma}, \boldsymbol{\sigma}\cdot\boldsymbol{\lambda}) = 0$. For this case, equation (20) is therefore very greatly simplified, giving

$$W^2 = c^4m_2(m_2 - 8m_2^0\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma})$$

or

$$W = \pm c^2\sqrt{m_2(m_2 + 16m_2^0)} \quad \text{spin } {}^1/_2 \quad (\uparrow\downarrow),$$

$$W = \pm c^2\sqrt{m_2(m_2 - 8m_2^0)} \quad \text{spin } {}^3/_2 \quad (\uparrow\uparrow).$$

This result is similar to that obtained in the spin 0, 1 case. For $m_2 = m_e$ and $m_2^0 = 2672\ m_e$, we obtain a stable charged particle of spin $^1/_2$, mass equal to that of the muon. If coupled electromagnetically to the highly unstable state of spin $^3/_2$, the spin $^1/_2$ state would be quenched, and, as in the case of spin 0, the natural decay caused by emission and absorption of a virtual photon would be enhanced by a sufficiently strong external field.

Eigenstates for the above cases are now found to be given by

$$\begin{pmatrix} \alpha & -i & 1 & 0 & 1 & i \\ i & \alpha & -i & -1 & 0 & 1 \\ 1 & i & \alpha & i & -1 & 0 \\ 0 & 1 & i & \beta & i & -1 \\ -1 & 0 & 1 & -i & \beta & i \\ i & -1 & 0 & -1 & -i & \beta \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_{13} \\ \psi_{24} \\ \psi_{25} \\ \psi_{36} \end{pmatrix} = 0,$$

$$\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma} = \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix}, \qquad \boldsymbol{\sigma}\cdot\boldsymbol{\lambda} = \begin{pmatrix} 0 & ix \\ -ix & 0 \end{pmatrix},$$

$$x = \begin{pmatrix} 0 & -i & 1 \\ i & 0 & -i \\ 1 & i & 0 \end{pmatrix},$$

together with the equations obtained by reversing the sign of W or S_z. Here,

$$\alpha = \frac{W - m_2c^2}{4m_2^0}, \qquad \beta = \frac{W + m_2c^2}{4m_2^0},$$

$$\alpha\beta - 2\beta + 2\alpha = 0 \qquad (\text{spin } ^1/_2),$$

$$\alpha\beta + \beta - \alpha = 0 \qquad (\text{spin } ^3/_2).$$

For the spin $^1/_2$ case, a normalized eigenfunction is

$$\frac{1}{\sqrt{6(m_\mu^2 + m_e^2)}} \begin{pmatrix} m_\mu + m_e \\ -i(m_\mu + m_e) \\ -(m_\mu + m_e) \\ i(m_\mu - m_e) \\ (m_\mu - m_e) \\ -i(m_\mu - m_e) \end{pmatrix} \exp\,(-im_\mu c^2 t/\hbar),$$

where we have written $W = m_\mu c^2$, $m_2 = m_e$. The opposite sign of the charge is automatically associated with the opposite sign of W. On the other hand, for the case $m_2 = 0$, this spin $^1/_2$ particle has zero charge and rest mass and represents another type of neutrino ($\uparrow \Downarrow$) which has a stable spin $^3/_2$ counterpart ($\uparrow \Uparrow$). Thus, for $m_2 = m_e$, $m_2 = 0$, equation (15) describes an electron and neutrino in the 1×1 representation of the β_i, and, in the 10×10 representation, particles that could be interpreted as a charged muon coupled similarly to a more complicated neutrino. These particles, like those of spin 1 or 0 discussed in the last section, are described by equations which are not irreducible representations of the Lorentz group, and in this sense such particles are not "elementary."

Rows and columns C, D of the 10×10 representation for the β_i lead to the eigenvalue equation

$$\begin{pmatrix} -\alpha, & -1, & -i, & 1 \\ -1, & \beta, & i, & -1 \\ i, & -i, & \beta, & i \\ 1, & -1, & -i, & \beta \end{pmatrix} \begin{pmatrix} \psi_{20} \\ \psi_{27} \\ \psi_{28} \\ \psi_{39} \end{pmatrix} = 0, \tag{21}$$

together with the equations obtained by changing the sign of W and/or S_z. In this subspace,

$$S_z = {}^1/_2\hbar \begin{pmatrix} -1, & 0, & 0, & 0 \\ 0, & 1, & -2i, & 0 \\ 0, & 2i, & 1, & 0 \\ 0, & 0, & 0, & -1 \end{pmatrix}; \qquad \boldsymbol{\sigma}\cdot\boldsymbol{\Sigma} = \begin{pmatrix} 0, & 0, & 0, & 0 \\ 0, & 0, & -i, & 1 \\ 0, & i, & 0, & -i \\ 0, & 1, & i, & 0 \end{pmatrix}$$

Two spin ${}^3/_2$ states ($\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma} = 1$, $S_z = 3\hbar/2$, $-\hbar/2$) which are also eigenstates of the charge operator $2\beta_4{}^2 - 1$ are given by

$$\psi_{3/2,\,3/2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix} e^{-iMc^2t/h}, \qquad \psi_{3/2,\,-1/2} = \frac{1}{\sqrt{6}} \begin{pmatrix} 0 \\ 1 \\ -i \\ 2 \end{pmatrix} e^{-iMc^2t/h},$$

where $\beta = 1$ or $W/c^2 = M = -m_2 + 4m_2{}^0$. With the above values of m_2 and $m_2{}^0$, this is a very heavy charged particle ($\sim 10^4\, m_e$) for which no direct evidence exists. However, it is coupled to a highly unstable particle given by $\alpha\beta + 2\alpha + 3 = 0$, the other solution of (21), or $W = 4m_2{}^0(-1 \pm \sqrt{2}i)$. This particle has spin 1/2 ($\boldsymbol{\sigma}\cdot\boldsymbol{\Sigma} = -2$) and it is described by the spinor

$$\psi_{1/2,\,-1/2} = \begin{pmatrix} (1-\beta) \\ -(\alpha+1) \\ i(\alpha+1) \\ (\alpha+1) \end{pmatrix} \exp\left(\frac{iMc^2t}{\hbar}\right) \exp\left(\pm\sqrt{2}\,\frac{Mc^2t}{\hbar}\right).$$

Finally, we note that the 5×5 representation of the β_i also leads to equation (21) together with the solution $\beta = 0$, or $W = m_ec^2$. Thus, in this representation, the above particle is coupled to electrons rather than muons. Hence, if states described by (21) were to exist in nature, they would decay rapidly into leptons and gamma-rays with the release of 5 Bev.

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